Lebanese American University Department of Computer Science and Mathematics

MTH207 – Discrete Structures 1

Exam 1 - Fall 2014

Exam Duration: 75 minutes

Name:

<u>ID#:</u>

<u>Instructions:</u> This exam consists of 9 problems and a total of 8 pages (last page is for scratch). Make sure no problem/page is missing. Answer the questions in the space provided for each problem. If more space is needed, you can use the back of the pages. To receive full credits, you have to justify your answers.

Question Number	Grade
1. (10%)	06
2. (8%)	08
3. (12%)	12
4. (10%)	P
5. (10%)	10
6. (12%)	12
7. (15%)	15
8. (15%)	15
9. (8%)	28
TOTAL	(96)

1. (10%) Show that m is a multiple of 3 if and only if m^2 is a multiple of 3.

show M= 3K (= m2=3K

first show: m=3k => m2=3k lef m= 3k; m2 = 3(3k2) V

let m2=3k, second, show: m2=3k => m=3k

12-34

contra-positive: 7(m=3k) => 7(m2=3k)

by contrapositive let n = 3k

let $m \neq 3k$ $\implies m^2 \neq 3k^2$ $\implies m^2 \neq 3(3k^2)$ $\implies m^2 \neq 3(3k^2)$ $\implies m^2 \neq 3(3k^2)$ True.

2. (8%) Show that $\sqrt{3}$ is an irrational number.

if v3 is rational them it can be written as m where m & n are two notward integers (n)0), in in simplest form (nordivisors).

 $\sqrt{3} = \frac{m}{n} \Rightarrow 3 = \frac{m^2}{n^2} \Rightarrow 3n^2 + m^2 \Rightarrow m^2$ is demultiple of $3 \Rightarrow m^2 = 3k$

DA21 >A21

me a here com buch.

" forst and tron
is wrong

3. (12%) Show that $((p \lor q) \land -p) \rightarrow q$ is a tautology using two different methods.

$$\frac{f}{|q|-|p|} \frac{|q|}{|p|} \frac{|p|}{|p|} \frac$$

$$(p \vee q) \wedge -p = (p \vee -p) \wedge (q \vee p) \longrightarrow q$$

$$= (p \wedge -p) \vee (p \wedge q) \longrightarrow q$$

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$$= (p \vee p) \wedge (q \vee p) \wedge (q \vee p) \wedge (q$$

$$(p \vee q) \wedge -p) = (p \vee -p) \wedge (q \vee p) \longrightarrow$$

$$= \frac{4}{\sqrt{4}}\sqrt{4}$$

$$= \frac{4}{\sqrt{4}}\sqrt{4}$$

$$|(p \vee q) \wedge -p| = (p \wedge -p) \vee (p \wedge q) -$$

$$= (p \wedge q) - s q$$

- \checkmark 4. (10%) Show by induction that $1+4+4^2+....+4^n=\frac{4^{n+1}-1}{3}$ where *n* is a non-negative integer (n = 0, 1, 2, ...)
 - · try for = 0; 1= 4-1=3

 - o assure $1+4+4^2+...+4^{n+1}=\frac{4^{n+1}-1}{3}$ is Tre.(n=0,1,2,...)Show $1+4+4^2+...+4^{n+4}+4^{n+1}=\frac{4^{n+2}-1}{3}$
 - 4nt = 4nt -1 4nt R.H. 4nt (421) = 4nt True

- 5. (10%) Show by induction that $2^{2n} 1$ is divisible by 3 for any positive integer $n \ge 1$.
- Try for n=1 & 22-1 = 4-1=3
- . Assume 2h1=3k for any tre integer ngl
- · show 22-1=3k for my +reinteger n>1
 - 2.2°-1=3k; 2°=3k+1 2(2°-1)-2
 - 2(3k+1)-1= 6K+1=
 - 2-20-1 = 1(20-12)=
 - 2-27-1-4-4-
- - = 3k(2-1)
 - = 3k Jac
- 2 1-1-2(21-1)-1
 - = 2(3k)-1

2(3K-1)

$$\frac{5}{2^{2n+2}} = 3k$$

$$(2.2^{2n}) = 4(2^{2n} - 1) - 3$$

$$= 2(3k) - 3$$

$$= 3(2k) - 3$$

$$= 3(2k - 1) \text{ Twen}$$



- **6.** Recall that n! = 1.2.3...n; for example 1! = 1;2! = 2;3! = 6;4! = 24;5! = 120;6! = 720..etc...Let P(n) represent the statement: P(n):1.1!+2.2!+3.3!+...+n.n!
 - a. (4%) Calculate P(1), P(2), P(3), P(4)

$$P(1) = 1$$
 $P(2) = 5$
 $P(3) = 1 + 4 + 18 = 23$
 $P(4) = \frac{23 + 24}{2} = 47$
 $= \frac{23 + 4}{2} = \frac{23 + 96}{2} = 119$

- 4
- b. (2%) Conjecture a formula for P(n)

$$P(n) = (n+1)!_{n} - 1$$



c. (6%) Prove your conjecture by mathematical induction.

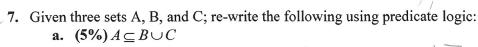
$$\text{show} \quad P(n+1) = (n+2)! = 1 = 1.1! + 2.2! + 3.3! + \cdots + n.h! + (n+1)(n+1)!$$

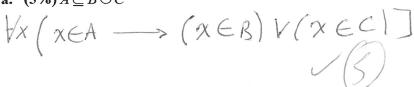
$$(n+1)! - 1 + (n+1)(n+1)!$$

R.HS=
$$(n+1)!(1+n+1)-1$$

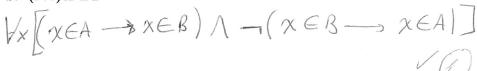
= $(n+1)!(n+2)-1$
= $(n+2)!-1$







b. (5%)
$$A \subset B$$

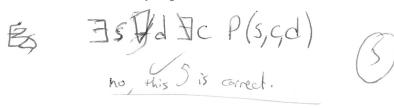


c.
$$(5\%) A = B$$



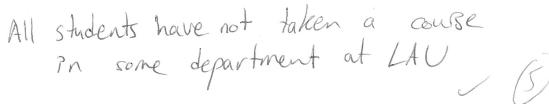


- 8. Let s denote the students at LAU, c the courses offered at LAU, d the departments at LAU, and P(s,c,d) denote that student s has taken course c in department d.
 - a. (5%) Express the following using quantifiers: There is a student who has taken some course in every department at LAU.



b. (5%) Using quantifiers, write the negation of the one you found in part a.

c. (5%) Express the negation in part b using English.



9. (8%) Let A, B, and C denote three sets. Show the following: If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then B = C.

if B=C => BCC and CCB

1) B CC . X E R . X E C . X E C . X E C . X E C . X E C . X E C . X E C . X E A; since ANB=ANC . X E R . X E A since AUB=AUC . X E C .